STAT 468

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Meta-Analytics: Tools for Understanding the Statistical Properties of Sports Metrics

Prof. Samuel Wong

Key reference

Alexander Franks, Alexander D'Amour, Daniel Cervone, Luke Bornn (2016). Meta-analytics: tools for understanding the statistical properties of sports metrics. *JQAS*, 12(4), 151–165.

Code: `https://github.com/afranks86/meta-analytics`

Why meta-analytics?

- In many sports, a myriad of metrics can be computed for each player
  - *Hockey*: Goals, Assists, Points, $\pm$, TOI, Shooting%, Hits, Blocks, Corsi, Fenwick, ...
  - *Basketball*: Field goals, 3-pointers, Free throws (attempts and %), Minutes played, Rebounds, Steals, Box plus minus, Value over replacement, ...
- How useful are these metrics? E.g.,
  - For evaluating a player's performance in a given season
  - For assessing the longer-term value of a player
- What makes a metric "good"?

Why meta-analytics?

- Meta-analytics or "meta-metrics" seek to ascertain which metrics provide the most unique and reliable information for decision-makers.
- From a statistical point of view: separate signal (e.g., player's ability) from noise, try to decompose observed variability into different sources.
- Adopt the context that most metrics are computed by player and season.

A mixed-effects model

Let X_{spm} be value of metric m for player p from season s.

A linear mixed-effects model might write:

$$X_{spm} = \mu_m + Z_{sm} + Z_{pm} + Z_{spm} + \epsilon_{spm}$$

where

$$Z_{sm} \sim [0, \sigma^2_{\text{SM}}]$$
$$Z_{pm} \sim [0, \sigma^2_{\text{PM}}]$$
$$Z_{spm} \sim [0, \sigma^2_{\text{SPM}}]$$
$$\epsilon_{spm} \sim [0, \tau^2_{\text{M}}]$$

$[\mu, \sigma^2]$ represents a distribution with mean μ and variance σ^2

A mixed-effects model

$$X_{spm} = \mu_m + Z_{sm} + Z_{pm} + Z_{spm} + \epsilon_{spm}$$

- Z_* are random effects, ϵ_{spm} as usual accounts for sampling variability unexplained by the regression
- Total variance: $Var(X_{spm}) = \sigma^2_{\text{SM}} + \sigma^2_{\text{PM}} + \sigma^2_{\text{SPM}} + \tau^2_{\text{M}}$
- Between-player, within-season variance: $\sigma^2_{\text{PM}} + \sigma^2_{\text{SPM}} + \tau^2_{\text{M}}$
- Between-season, within-player variance: $\sigma^2_{\text{SM}} + \sigma^2_{\text{SPM}} + \tau^2_{\text{M}}$

A mixed-effects model

More generally, let X be a r.v. representing a randomly chosen season/player/metric combination X_{spm}.

Then look at conditional expectations and variances such as

$$
\begin{aligned}
E_{spm}[X] &= E[X|S = s, P = p, M = m] \\
V_{spm}[X] &= Var[X|S = s, P = p, M = m] \\
E_{sm}[X] &= E[X|S = s, M = m] \\
V_{sm}[X] &= Var[X|S = s, M = m] \\
E_{m}[X] &= E[X|M = m] \\
V_{m}[X] &= Var[X|M = m]
\end{aligned}
$$

Meta-metric 1: Discrimination

- A metric's ability to discriminate between different players, i.e., the focus is between-player variance.
- Of the total between-player variance in season s, what fraction is due to chance and what fraction is due to signal?
- Since the law of total variance gives

$$V_{sm}[X] = E_{sm}[V_{spm}[X]] + V_{sm}[E_{spm}[X]]$$

- The discriminative power of a metric m in season s can be defined as

$$\mathcal{D}_{sm} = 1 - \frac{E_{sm}[V_{spm}[X]]}{V_{sm}[X]}$$

Meta-metric 1: Discrimination

- In the mixed-effects model,

$$\mathcal{D}_{sm} = 1 - \frac{\tau^2_{\text{M}}}{\sigma^2_{\text{PM}} + \sigma^2_{\text{SPM}} + \tau^2_{\text{M}}}$$
$$= \frac{\sigma^2_{\text{PM}} + \sigma^2_{\text{SPM}}}{\sigma^2_{\text{PM}} + \sigma^2_{\text{SPM}} + \tau^2_{\text{M}}}$$

- In general, can estimate via

$$\hat{\mathcal{D}}_{sm} = 1 - \frac{\frac{1}{P}\sum_{p=1}^{P} \text{BV}[X_{spm}]}{\frac{1}{P}\sum_{p=1}^{P}(X_{spm} - \bar{X}_{s\cdot m})^2}$$

where BV is the bootstrap variance of $V_{spm}[X]$.

Meta-metric 2: Stability

- How much does an individual player's metric vary from season to season?
- Stable metric provides confidence that this year's performance will be predictive of next year's performance.
- Define *stability* as:

$$\mathcal{S}_m = 1 - \frac{E_m[V_{pm}[X] - V_{spm}[X]]}{V_m[X] - E_m[V_{spm}[X]]}$$

i.e., 1− ratio of "average between-season variance" to "total variance" (excluding sampling variability)

Meta-metric 2: Stability

- In the mixed-effects model,

$$\mathcal{S}_m = 1 - \frac{\sigma^2_{\text{SM}} + \sigma^2_{\text{SPM}} + \tau^2_{\text{M}} - \tau^2_{\text{M}}}{\sigma^2_{\text{PM}} + \sigma^2_{\text{SM}} + \sigma^2_{\text{SPM}} + \tau^2_{\text{M}} - \tau^2_{\text{M}}}$$
$$= \frac{\sigma^2_{\text{PM}}}{\sigma^2_{\text{PM}} + \sigma^2_{\text{SM}} + \sigma^2_{\text{SPM}}}$$

- In general, can estimate via

$$\hat{\mathcal{S}}_m = 1 - \frac{\frac{1}{P}\sum_{p=1}^{P}\frac{1}{S_p}\sum_{s=1}^{S_p}\left[(X_{spm} - \bar{X}_{\cdot pm})^2 - \text{BV}[X_{spm}]\right]}{\frac{1}{P}\sum_{p=1}^{P}\frac{1}{S_p}\sum_{s=1}^{S_p}\left[(X_{spm} - \bar{X}_{\cdot\cdot m})^2 - \text{BV}[X_{spm}]\right]}$$

Meta-metric 3: Independence

- Since there are so many possible metrics, how much "new" or independent information does a given metric provide?
- Say there's M metrics of interest, let Z_{sp} be the M-vector of metrics for player p during season s. Use a Normal copula model:

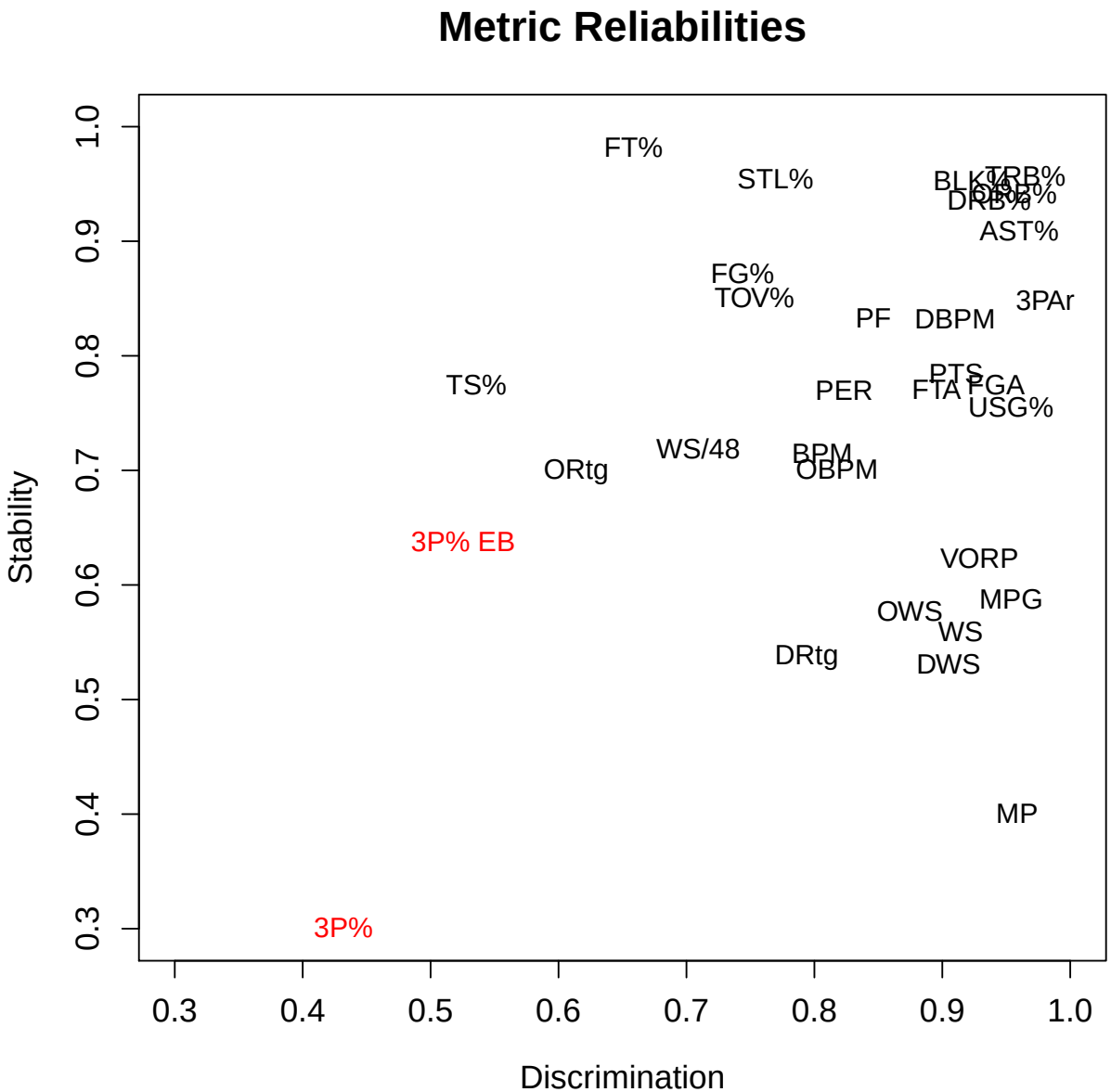
$$Z_{sp} \overset{iid}{\sim} \text{MVN}(0, C)$$
$$X_{spm} = F_m^{-1}[\Phi(Z_{spm})],$$

- Then define *independence* of a metric m given a set of other metrics $\mathcal{M}$, as

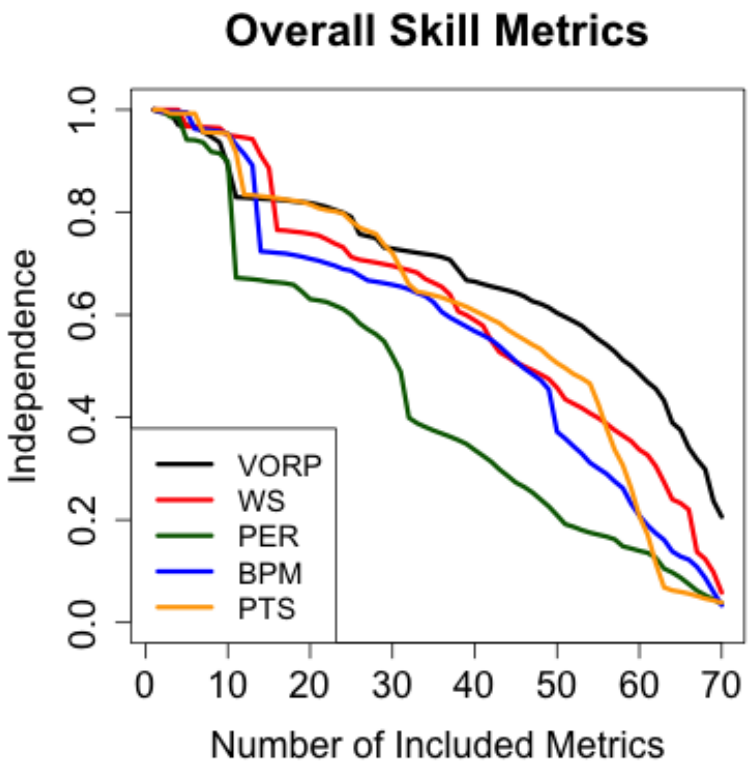
$$\mathcal{I}_{m\mathcal{M}} = \frac{Var\left[Z_{spm} \mid \{Z_{spq} : q \in \mathcal{M}\}\right]}{Var[Z_{spm}]}$$

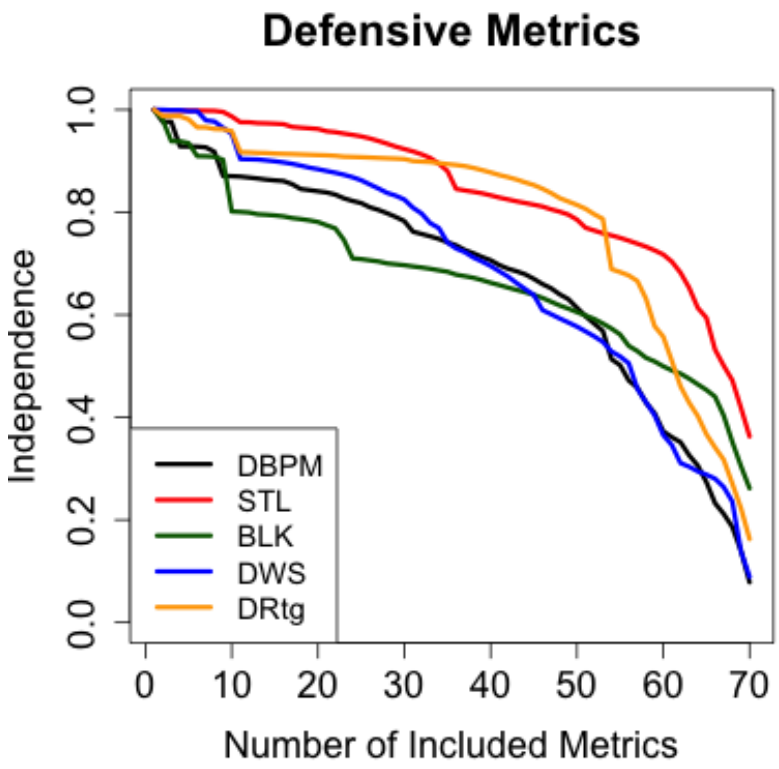
  which has a nice R-squared interpretation.
- Principal component analysis on C is a complementary tool

Results: NBA

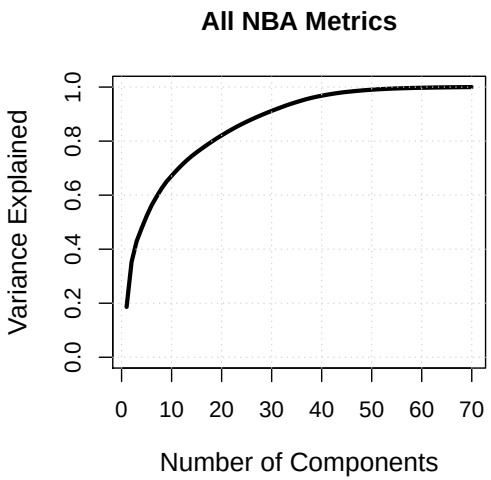

Metric Reliabilities
Stability
Discrimination
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
FT%
STL%
TRB%
DRB%
AST%
FG%
TOV%
3PAr
PF
DBPM
PTS
TS%
PER
FTA
FGA
USG%
WS/48
BPM
ORtg
OBPM
3P% EB
VORP
MPG
OWS
WS
DRtg
DWS
MP
3P%

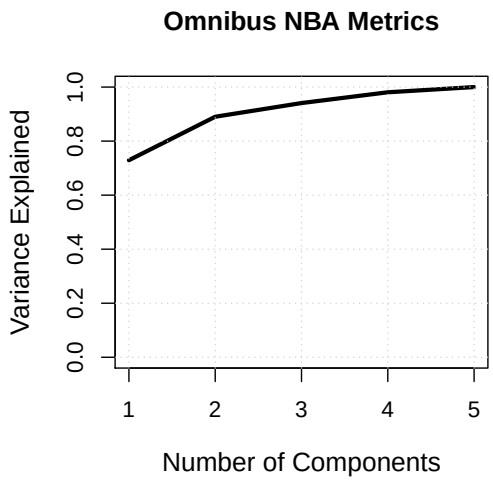
Results: NBA

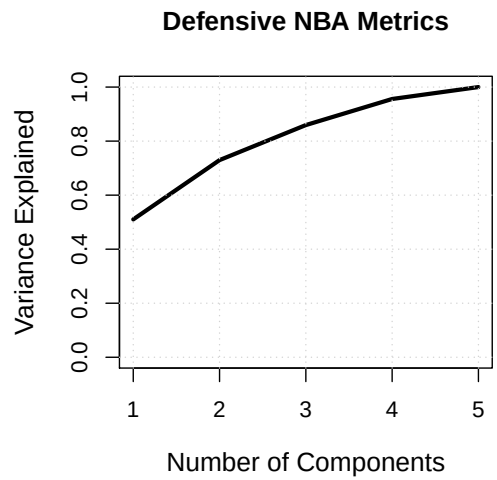

Overall Skill Metrics
Independence
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60
70
Number of Included Metrics
VORP
WS
PER
BPM
PTS


Defensive Metrics
Independence
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60
70
Number of Included Metrics
DBPM
STL
BLK
DWS
DRtg

Results: NBA


All NBA Metrics
Variance Explained
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60
70
Number of Components

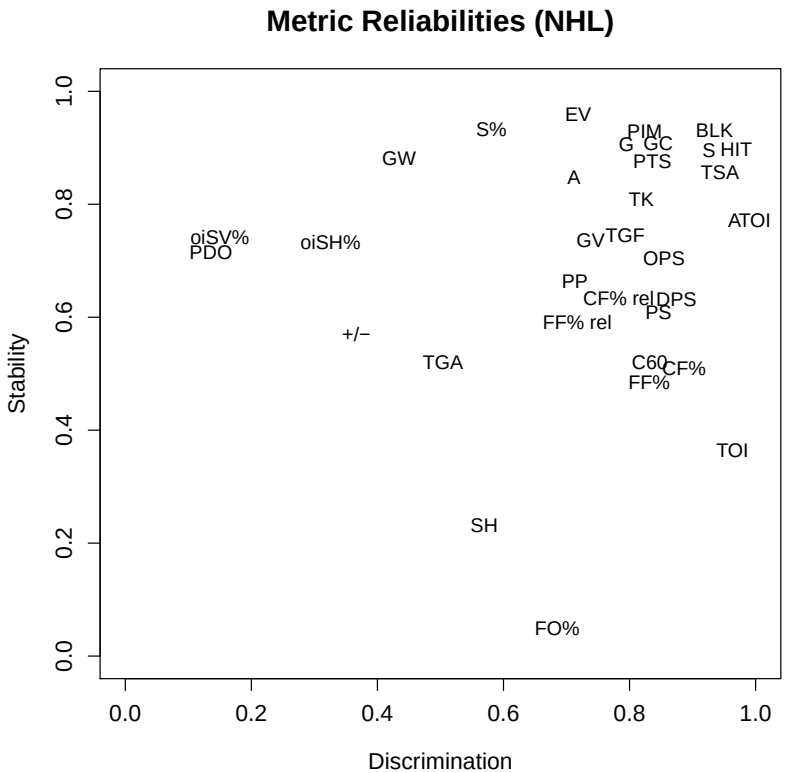

Omnibus NBA Metrics
Variance Explained
0.0
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
Number of Components


Defensive NBA Metrics
Variance Explained
0.0
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
Number of Components

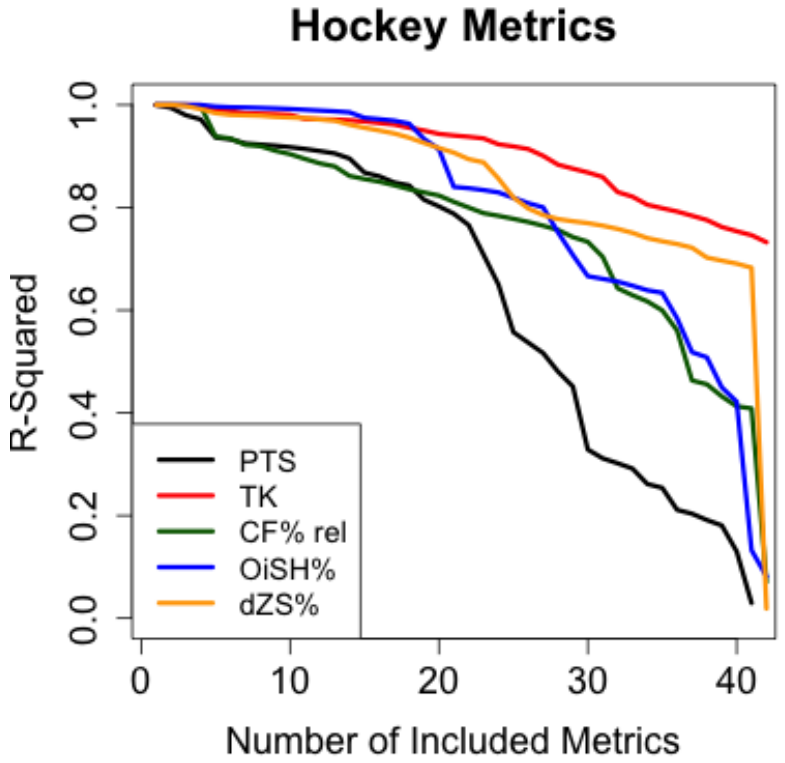
NBA metric (partial) list

Metric	Description
MP	Minutes played
FGA	Field goal attempts
FG%	Field goal percentage
3PA	3 point attempts
3P%	3 point percentage
2PA	2 point attempts
2P%	2 point percentage
FTA	Free throw attempts
FT%	Free throw percentage
PF	Personal fouls
PTS	Points
PER	Personal efficiency rating
TS%	True shooting percentage
3PAr	Three point attempt rate
FTr	Free throw attempt rate
ORB	Offensive rebounds
DRB	Defensive rebounds
TRB	Total rebounds
AST	Assists
STL	Steals
BLK	Blocks
TOV%	Turnover percentage (per possession)
USG%	Usage percentage
OWS	Offensive win shares
DWS	Defensive win shares
WS	Win shares
WS/48	Win shares per 48 minutes
OBPM	Offensive box plus minus
DBPM	Defensive box plus minus
BPM	Box plus minus
VORP	Value over replacement
ORtg	Offensive rating
DRtg	Defensive rating
Avg Shot Dist	Average shot distance

Results: NHL

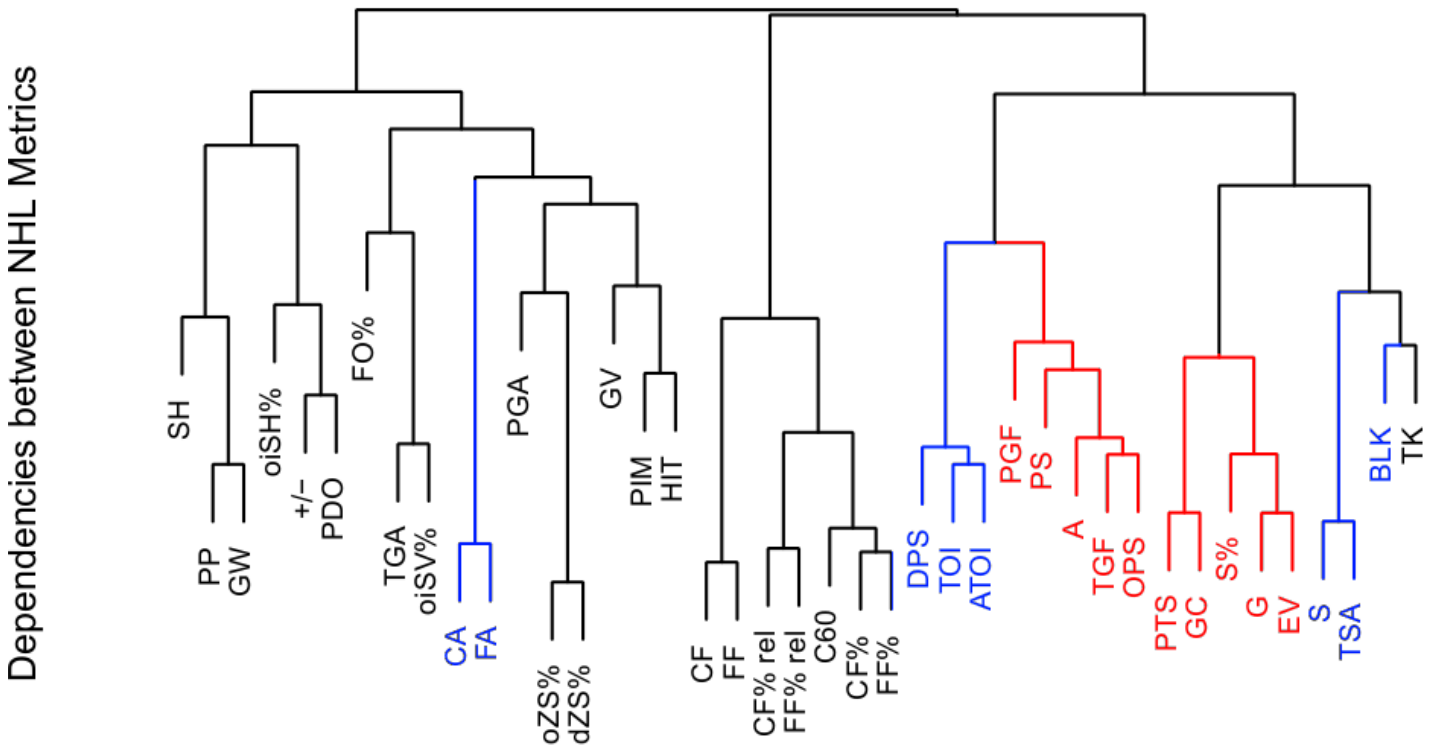


(a)



(b)

Results: NHL


Dependencies between NHL Metrics
SH
PP
GW
oiSH%
+/−
PDO
FO%
TGA
oiSV%
CA
FA
PGA
oZS%
dZS%
GV
PIM
HIT
CF
FF
CF% rel
FF% rel
C60
CF%
FF%
DPS
TOI
ATOI
PGF
PS
A
TGF
OPS
PTS
GC
S%
G
EV
S
TSA
BLK
TK

NHL metric list

Metric	Description
G	goals
A	assists
PTS	points
±	plus / minus
PIM	penalties in minutes
EV	even strength goals
PP	power play goals
SH	short handed goals
GW	game winning goals
S	shots on goal
S%	shooting percentage
TSA	total shots attempted
TOI	time on ice
FO%	face off win percentage
HIT	hits at even strength
BLK	blocks at even strength
TK	takeways
GV	giveaways
GC	goals created
TGF	total goals for (while player was on the ice)
PGF	power player goals for (while player was on the ice)
TGA	total goals against (while player was on the ice)
PGA	power player goals against (while player was on the ice)
OPS	offensive point shares
DPS	defensive point shares
PS	total point shares

NHL metric list

Metric	Description
CF	Corsi for (on ice shots+blocks+misses)
CA	Corsi against (on ice shots+blocks+misses)
CF%	Corsi for percentage: CF / (CF + CA)
CF% rel	Relative Corsi for (on ice CF% - off ice CF%)
FF	Fenwick for (shots+blocks+misses)
FA	Fenwick against (shots+blocks+misses)
FF%	Fenwick for percentage: FF / (FF + FA)
FF% rel	Relative Fenwick for (on ice FF% - off ice FF%)
oiSH%	Team on ice shooting percentage while player on the ice
oiSV%	Team on ice save percentage while player on the ice
PDO	Shooting percentage plus save percentage
oZS%	percentage of offensive zone starts while on the ice
dZS%	percentage of defensive zone starts while on the ice

Discussion

- Other applications: designing new metrics to be more stable, discriminative, or independent
- Consider subsets/groups of players, since player type/position explains a lot of variability ("conditional meta-metrics")
- Merge these quantitative characterizations with qualitative "relevance" from domain knowledge